DS3 Second degré
CORRECTION

Exercice 1 (8)

1) x^2-2x+4 $a=1$ $b=-2$ $c=4$

$\Delta = b^2-4ac = (-2)^2-4\times1\times4 = -12<0$ 0,25

Donc pas de factorisation possible 0,25 (0,5)

2) $A(x) = -x^2+2x+6$ $a=-1$ $b=2$ $c=6$

$\Delta = 2^2-4\times(-1)\times6 = 28>0$ 0,5 donc 2 racines

$x_1 = \dfrac{-b-\sqrt{\Delta}}{2a} = \dfrac{2-\sqrt{28}}{2\times(-1)} = 1+\sqrt{7}$ $x_2 = \dfrac{-b+\sqrt{\Delta}}{2a} = 1-\sqrt{7}$ 0,5

ici $a=-1<0$ donc

(1,5)

x	$-\infty$		$1-\sqrt{7}$		$1+\sqrt{7}$		$+\infty$
signe de A		$-$	0	$+$	0	$-$	

0,5

$B(x) = 2x^2-4x+2$ $a=2$ $b=-4$ $c=2$

$\Delta = (-4)^2-4\times2\times2 = 0$ 0,5 donc 1 racine

$x_0 = -\dfrac{b}{2a} = \dfrac{4}{2\times2} = 1$ 0,5

(1,5)

x	$-\infty$		1		$+\infty$
signe de B		$+$	0	$+$	

0,5

3) $A(x) = -x^2 + 2x + 6$ $\quad a = -1 \quad b = 2 \quad c = 6$

$\alpha = \frac{-b}{2a} = \frac{-2}{2\times(-1)} = 1$ $\qquad \beta = A(\alpha) = A(1) = -1^2 + 2\times 1 + 6$

$a = -1$ donc $\cap$ $\qquad \beta = 7$

Donc

x	$-\infty$	1	$+\infty$
variations de A	$\nearrow$	7	$\searrow$

1,5/

$B(x) = 2x^2 - 4x + 2$ $\quad a = 2 \quad b = -4 \quad c = 2$

$\alpha = \frac{-b}{2a} = \frac{4}{2\times 2} = 1$ $\qquad \beta = B(1) = 2\times 1^2 - 4\times 1 + 2$

$a = 2 > 0$ donc $\cup$ $\qquad \beta = 0$

x	$-\infty$	1	$+\infty$
variations	$\searrow$	0	$\nearrow$

1,5/

4) $2x^2 - 2x - 5 \leq 2x - 25$

$2x^2 - 2x - 2x - 5 + 25 \leq 0$ donc $2x^2 - 4x + 20 \leq 0$

$a = 2 \quad b = -4 \quad c = 20$

$\Delta = (-4)^2 - 4\times 2\times 20 = - \quad < 0$

1,5/

Donc

x	$-\infty$	$+\infty$
signe de $2x^2 - 4x + 20$	$+$	

$\boxed{S = \emptyset}$

Exercice 2

4

$f(x) = 4x^3 - 18x^2 + 16x - 4$

1) $(2x-1)(ax^2+bx+c) = 2ax^3 + 2bx^2 + 2cx$
$-ax^2 - bx - c$

$= 2a\,x^3 + (2b-a)x^2 + (2c-b)x - c$

Par identification : $4 = 2a$ donc $a = 2$

$-18 = 2b - a$
$-18 = 2b - 2$
$-18 + 2 = 2b$
$-16 = 2b$ $\quad b = -8$

$16 = 2c - b$
$16 = 2c + 8$
$8 = 2c$ $\quad c = 4$

vérification : $-4 = -c$
$c = 4$ OK.

Donc $\boxed{f(x) = (2x-1)(2x^2 - 8x + 4)}$ 1/

2) $f(x) = 0 \iff (2x-1)(2x^2-8x+4) = 0$

un produit de facteurs est nul si et seulement si un des facteurs est nul donc:

$2x - 1 = 0$	$2x^2 - 8x + 4 = 0$
$2x = 1$	$a = 2 \quad b = -8 \quad c = 4$
$x = \frac{1}{2}$	$\Delta = b^2 - 4ac$
	$\Delta = (-8)^2 - 4 \times 2 \times 4$
	$\Delta = 32 > 0$
	$x_1 = 2 - \sqrt{2}$
	$x_2 = 2 + \sqrt{2}$

1,5/

3)

x	$-\infty$	$\frac{1}{2}$		$2-\sqrt{2}$		$2+\sqrt{2}$	$+\infty$
$2x-1$	$-$	0	$+$	|	$+$	|	$+$
$2x^2-8x+4$	$+$	|	$+$	0	$-$	0	$+$
signe de f	$-$	0	$+$	0	$-$	0	$+$

1,5/

Exercice 3

$f(x) = 2x^2 + 4x - 6$ $a = 2$ $b = 4$ $c = -6$

$g(x) = 3x^2 - 4x + m + 1$

1) $f(1) = 2 \times 1^2 + 4 \times 1 - 6 = 2 + 4 - 6 = 0$

Donc 1 est racine évidente de f. $x_1 = 1$

Or $S = -\frac{b}{a}$ $x_1 + x_2 = \frac{-4}{} = -2$

$x_1 + x_2 = -2$ $1 + x_2 = -2$ $x_2 = -2 - 1$

$x_2 = -3$ $\boxed{\text{Les racines sont } -3 \text{ et } 1}$

2) BONUS $h(x) = a(x - x_1)(x - x_2)$

$h(x) = a(x+1)(x-2)$

or $h(1) = -4$ donc $h(1) = a(1+1)(1-2) = -4$

$a \times 2 \times (-1) = -4$ $-2a = -4$ $a = 2$

donc $\boxed{h(x) = 2(x+1)(x-2)}$ $= 2(x^2 + x - 2x - 2)$

$\boxed{h(x) = 2x^2 - 2x - 4}$

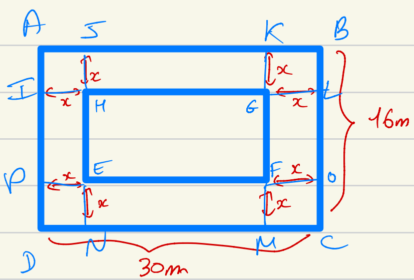
3) $g(-1) = 0$

donc $3\times(-1)^2 - 4\times(-1) + 4m + 1 = 0$

$3 + 4 + 4m + 1 = 0 \quad 4m + 8 = 0 \quad 4m = -8$

$\boxed{m = -2}$

Exercice 4



1) $\mathcal{A}(\text{ruelle}) = \mathcal{A}(\text{p vegetale})$

$\mathcal{A}(\text{ruelle}) = \mathcal{A}(EFGH)$

$\mathcal{A}(EFGH) = EH \times EF = (16-2x)(30-2x)$

$\mathcal{A}(EFGH) = 480 - 32x - 60x + 4x^2$

$\boxed{\mathcal{A}(EFGH) = 4x^2 - 92x + 480}$

$\mathcal{A}(\text{ruelle}) = \mathcal{A}(AILB) + \mathcal{A}(DCOP) + \mathcal{A}(IHEP) + \mathcal{A}(GLOF)$

$= x\times 30 + x\times 30 + x\times(16-x-x) + x\times(16-x-x)$

$= 30x + 30x + x(16-2x) + x(16-2x)$

$= 60x + 16x - 2x^2 + 16x - 2x^2$

$\boxed{\mathcal{A}(\text{ruelle}) = -4x^2 + 92x}$

Donc on obtient :

$$4x^2 - 92x + 480 = -4x^2 + 92x$$

$$\Leftrightarrow \quad \underline{8x^2 - 184x + 480 = 0}$$

$a = 8 \quad b = -184 \quad c = 480$

$\Delta = b^2 - 4ac = (-184)^2 - 4 \times 8 \times 480$ /25

$\Delta = 18\,496 > 0$ donc 2 racines

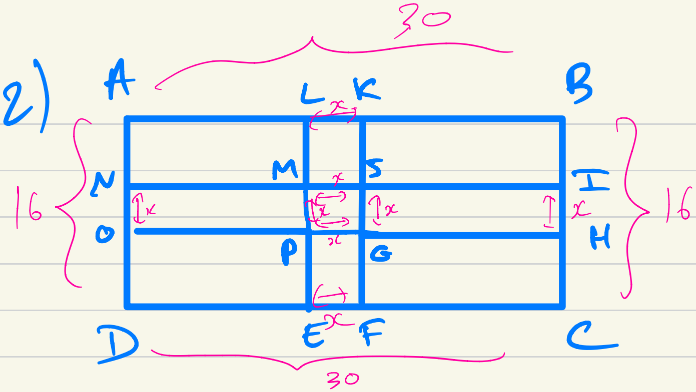
$$x_1 = \frac{-b - \sqrt{\Delta}}{2a} = \frac{184 - \sqrt{18496}}{2 \times 8} = 3$$

$$x_2 = \frac{-b + \sqrt{\Delta}}{2a} = \frac{184 + \sqrt{18496}}{2 \times 8} = 20$$

or $0 < x < 16$ donc $x = 3$

$\boxed{\text{La ruelle doit faire 3m de large}}$

2)



$$\mathcal{A}(\text{ruelle}) = \mathcal{A}(NIHO) + \mathcal{A}(LKFE) - \mathcal{A}(MJGP)$$

$$= 30 \times x + 16 \times x - x \times x$$

$$= 30x + 16x - x^2$$

$$\underline{\mathcal{A}(\text{ruelle}) = -x^2 + 46x}$$

$$\mathcal{A}(\text{végétal}) = \mathcal{A}(ALMN) + \mathcal{A}(KJIB) + \mathcal{A}(GHCF)$$

$$+ \mathcal{A}(DEPO) = \mathcal{A}(ALMN) \times 4$$

$$= AL \times AN = \frac{30-x}{2} \times \frac{16-x}{2}$$

$$= \frac{(30-x)(16-x)}{4} = \frac{480 - 30x - 16x + x^2}{4}$$

$$\underline{\mathcal{A}(\text{végétal}) = \frac{x^2 - 46x + 480}{4}}$$

Donc on obtient : $-x^2+46x = \dfrac{x^2-46x+480}{4}$

$(-x^2+46x)\times 4 = x^2-46x+480$

$-4x^2+184x-x^2+46x-480=0$

$-5x^2+230x-480=0$

$a=-5$ $\quad b=230$ $\quad c=-480$

$\Delta = 230^2-4\times(-5)\times(-480)$

$\Delta =$

/2

Rattrapage

1) x^2-4x+4 $\quad a=1 \quad b=-4 \quad c=4$

$\Delta = b^2-4ac = (-4)^2-4\times 1\times 4 = 0$

$a(x-x_0)^2$ $\qquad x_0 = \frac{-b}{2a} = \frac{4}{2} = 2$

$= 1(x-2)^2$

$= \boxed{(x-2)^2}$

2) $A(x) = -2x^2-4x+16$

$a=-2 \quad b=-4 \quad c=16$

$\Delta = (-4)^2-4\times(-2)\times 16$

$\Delta = 144>0$ 2 sols

$x_1 = \frac{4-12}{-4} = 2 \qquad x_2 = \frac{4+12}{-4} = -4$

x	$-\infty$		-4		2		$+\infty$
		$-$	0	$+$	0	$-$	

$B(x) = 5x^2+10x-15$

$a=5 \quad b=10 \quad c=-15$

$\Delta = 100-4\times 5\times -15$

$\Delta = 400>0$

$x_1 = \frac{-10-20}{10} = -3$

$x_2 = \frac{-10+20}{10} = 1$

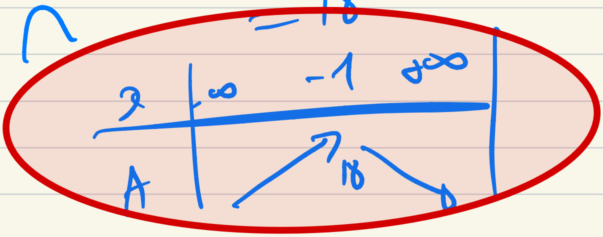
		-3		1	
	$+$	0	$-$	0	$+$

$\alpha = \frac{-b}{2a} = \frac{4}{2\times(-2)} = -1$

$\beta = f(-1) = A(-1)$
$= -2\times(-1)^2 - 4\times(-1) + 16$
$= -2 + 4 + 16$
$= 18$

$a = -2$

$\cap$

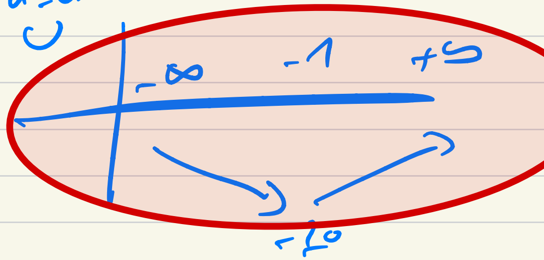


$\alpha = \frac{-b}{2a} = \frac{-10}{2\times 5} = -1$

$\beta = f(-1) = 5\times(-1)^2 + 10\times(-1) - 15$
$= 5 - 10 - 15$
$= -20$

$a = 5 > 0$

$\cup$

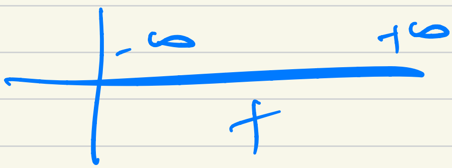


4) $2x^2 - 3x - 5 \geq x - 25$

$2x^2 - x - 3x - 5 + 25 \geq 0$

$2x^2 - 4x + 20 \geq 0$

$\Delta < 0$



$S = \mathbb{R}$

ex2 $f(x) = -36x^2 + 8x^3 + 32x - 8$

$$f(x) = 8x^3 - 36x^2 + 32x - 8$$

1) $f(x) = (2x-1)(ax^2+bx+c)$

$$= 2ax^3 + 2bx^2 + 2cx - ax^2 - bx - c$$

$$= 2ax^3 + (2b-a)x^2 + (2c-b)x - c$$

$2a = 8$	$2b - a = -36$	$-c = -8$
$\boxed{a = 4}$	$2b - 4 = -36$	$\boxed{c = 8}$
	$2b = -32$
	$\boxed{b = -16}$

1) $f(x) = (2x-1)(4x^2 - 16x + 8)$

2) $S = \left\{ \frac{1}{2} ; 2 - \sqrt{2} ; 2 + \sqrt{2} \right\}$

3)

x		$\frac{1}{2}$		$2-\sqrt{2}$		$2+\sqrt{2}$	
	$-$	0	$+$	0	$-$	0	$+$

exercice 3

$f(x) = 3x^2 - x - 10$ $\qquad a = 3 \quad b = -1 \quad c = -10$

$f(2) = 3 \times 2^2 - 2 - 10$ $\qquad$ 2 est solution

$f(2) = 12 - 2 - 10 = 0$

$S = -\frac{b}{a} = \frac{1}{3}$ $\qquad \frac{1}{3} = x_1 + x_2$

$\frac{1}{3} = 2 + x_2 \qquad x_2 = \frac{1}{3} - 2 = \frac{1}{3} - \frac{6}{3} = -\frac{5}{3}$

$S = \left\{ -\frac{5}{3} ; 2 \right\}$

exercice